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Light-front QCD : present status

A Harindranath*
Theory Group, Saha Institute of Nuclear Physics,
1/AF Bidhannagar, Calcutta-700 064, India

Abstract : We review the present status of light-front Hamiltonian approach to solve Quantum Chromodynamics (QCD). After providing a brief motivation for the use of light-front dynamics, we discuss a recently proposed similarity renormalization group approach to QCD We summarize recent advances made in the study of confinement in this approach The features of chiral symmetry breaking on the light-front are highlighted A new approach to the study of deep inelastic structure functions combining coordinate space and momemtum space techniques is briefly outlined Lastly we mention some of the open problems in the field.

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1. Why light-front ?

Light-front dynamics [1] was introduced by Dirac in 1949. He found that one may set up a dynamical theory in which the dynamical variables refer to physical conditions on a light-front $x^+ = x^0 + x^3 = 0$. x^+ is the light-front time and x^- is the light-front longitudinal space variable. Transverse variable $x^\perp = (x^1, x^2)$. For an on-mass shell particle, longitudinal momentum $k^+ = k^0 + k^3 \geq 0$ and energy $k^- = \frac{(k^\perp)^2 + m^2}{k^+}$. From this dispersion relation, we observe that large energy divergences occur from large $k^\perp$ and small k^+ and since they appear *not* additively in the expression for energy, one can expect nonlocal counterterms which results in a complex renormalization problem. Thus one may legitimately ask : why bother ?

To answer this question, we have to take a look at the symmetries of light-front. First consider the boosts. Under a longitudinal boost, $x^\pm \rightarrow e^{\pm\phi}x^\pm$. Thus longitudinal boost is simply a scaling operation which leaves $x^+ = 0$ invariant. In canonical field theory, generators of longitudinal boost and scale transformations obey identical commutation relations. Since longitudinal boost invariance is an exact Lorentz symmetry, it cannot be

*e-mail : hari@tnp.saha.ernet.in

violated by masses which is in sharp contrast to usual scale invariance. On the other hand, transverse boosts are exactly Galilean boosts familiar in non-relativistic dynamics which also leave $x^+ = 0$ invariant. The fact that boost symmetry on the light-front is kinematical has interesting consequences, for example, in the computation of the elastic form factor of composite systems [2].

Since only $x^\perp$ carry inverse mass dimension x^- and $x^\perp$ have to treated differently in the scaling analysis. It immediately follows that power counting is different on the light-front [3].

Next consider rotations. Rotations in the transverse plane are kinematical (light-front helicity is kinematical) whereas transverse rotations change $x^+ = 0$ and hence are dynamical and as complicated as Hamiltonian.

An attractive feature of the light-front is the apparent triviality of the vacuum. For a massive on-shell particle, $k^+ \geq 0$. On the other hand vacuum processes receive contributions only from $k^+ = 0$. If $k^+ = 0$ is removed (say, by imposing a cutoff $k_i^+ \geq \varepsilon$) then Fock space vacuum is an eigen state of the full Hamiltonian. Thus, to build a hadron we need not worry about the ground state of the theory. Thus the constituent picture of hadrons which underlies ever popular quark models of hadrons may find justification in quantum field theory.

From the dispersion relation $k^- = \frac{(k^\perp)^2+m^2}{\;}$, k^+ near ε which corresponds to long longitudinal distances along the light cone appears as ultraviolet (large) divergences in energy. This offers a possibility to address long distance effects (nonperturbative issues) through renormalization.

After this brief introduction to the features of light-front dynamics, we take a look at the canonical Hamiltonian of light-front QCD.

2. Light-front QCD

2.1. *Canonical structure :*

Choosing the gauge $A_a^+ = 0$, the canonical Hamiltonian of light-front QCD can be constructed from either the Lagrangian density or from light-front power counting.

$$H = \int dx^- d^2x_\perp \Bigg\{ \frac{1}{2}\left(\partial^i A_a^j\right)^2 + g f^{abc} A_a^i A_b^j A_c^j + \frac{g^2}{4} f^{abc} f^{ade} A_b^i A_c^j A_d^i A_e^j$$
$$+ \left[\xi^\dagger \{\sigma_\perp \cdot (i\partial_\perp + gA_\perp) - im_F\} \left(\frac{1}{i\partial^+}\right) \{\sigma_\perp \cdot (i\partial_\perp + gA_\perp) + im_F\} \xi\right]$$
$$+ g\partial^i A_a^i \left(\frac{1}{\partial^+}\right)\left(f^{abc} A_b^j \partial^+ A_c^j + 2\xi^\dagger T^a \xi\right) + \frac{g^2}{2}\left(\frac{1}{\partial^+}\right)$$
$$\times \left(f^{abc} A_b^i \partial^+ A_c^i + 2\xi^\dagger T^a \xi\right)\left(\frac{1}{\partial^+}\right)\left(f^{ade} A_d^j \partial^+ A_e^j + 2\xi^\dagger T^a \xi\right)\Bigg\}. \quad (1)$$

At the tree level itself, canonical Hamiltonian exhibits processes which are sensitive to k^+ near zero for gluons and processes sensitive to k^+ near zero for quarks.

2.2. Similarity renormalization approach :

To investigate the low energy structure, namely the bound state problem, one may visualize solving the eigen value equation

$$P^- |\Psi\rangle = \frac{M^2 + (P^\perp)^2}{P^+} |\Psi\rangle . \tag{2}$$

with the state vector $|\Psi\rangle$ expanded in terms of the multi-parton wave functions. Unfortunately this is a never ending series in field theory and direct diagonalization is too difficult to tackle. It is clear that one needs to make approximations. Any cutoff Hamiltonian necessarily violates the sacred (Lorentz and Gauge) symmetries of the theory and we have to figure out how to restore them. The important question is how to get finite answers that are sensible.

Similarity Renormalization group approach [3] to tackle this problem was introduced by Glazek and Wilson. Given the bare cutoff canonical Hamiltonian, to solve the bound state problem, a two-step process is devised. First, effects at relativistic momenta are computed using perturbation theory and possible structures of the counterterms are identified. Second, the effective Hamiltonian at an appropriate low energy scale is diagonalized to yield low energy observables. The effective Hamiltonian at the low energy scale is constructed from the bare cutoff Hamiltonian using a similarity transformation which is designed so that no vanishing energy denominators appear in every order of perturbation theory and the effective Hamiltonian does not cause transition between low energy and high energy states.

At the second step, by lowering the energy scale, particle degrees of freedom are eliminated in favor of effective interactions that do not change particle number. If we choose the energy scale to be just of the order of hadronic mass scale, the character of the bound state problem changes from a field theoretic computation with arbitrary number of constituents to a computation dominated by potentials. At that level, the coupling does not run, we can choose it to be weak, and model the bound state calculation after that of QED. By increasing the scale, we bring back relativistic processes and hope to get closer to QCD.

2.3. Alternatives :

Alternative methods with the same goal in mind have been devised in the past. The Discrete Light-Cone Quantization (DLCQ) program [4] of Brodsky, Pauli and collaborators attempts a direct discretization in momentum space (k^+, $k^\perp$). The transverse lattice Hamiltonian approach [5] of Bardeen, Pearson and Rabinovici treat x^+ and x^- continuous while treating the transverse space $x^\perp$ as discrete.

2.4. Confinement :

A second order analysis of processes sensitive to small k^+ gluon in the similarity renormalization (SR) scheme has lead to the emergence of logarithmic confinement [6]. Conventional perturbation theory leads to a complete cancellation of small k^+ divergences in the single quark self energy and one gluon exchange processes. But SR perturbation theory analysis leads to a partial cancellation. In an analysis with the small longitudinal momentum cutoff ($k^+ > \epsilon$) both contributions contain log ϵ plus finite terms. For color singlet states log ϵ terms cancels between the two type of processes. The remaining finite terms behave like log $|x^-|$ for large x^- and log $|x^\perp|$ for large $x^\perp$. Utilizing this confinement mechanism first principle calculations have been performed recently for the spectroscopy of heavy quark systems [7].

2.5. Chiral symmetry breaking :

For the cut off theory ($k^+ = 0$ mode removed) vacuum is trivial. This means mechanisms for the effects associated with spontaneous symmetry breaking are very different in this theory. Further, chiral symmetry is exact for free quarks of any mass which means that mechanisms for the effects associated with explicit breaking are also different.

The second statement above may appear rather strange for someone unfamiliar with the features of the light-front. On the light-front it turns out that chirality is simply helicity. The basic reason behind this remarkable property is the fact that on the light-front the four component fermion field can be decomposed as $\psi = \psi^+ + \psi^-$. The component ψ^+ is dynamical and ψ^- is constrained. In $A^+ = 0$ gauge the constraint relation is

$$\psi^-(x^-, x^\perp) = -\frac{i}{4}\int dy^- \epsilon(x^- - y^-)\left[\alpha^\perp \cdot (i\partial^\perp + gA^\perp) + \gamma^0 m\right] \times \psi^+(y^-, x^\perp). \tag{3}$$

The fermion mass enters the Hamiltonian only through ψ^-. Introducing the two component field η

$$\psi_+ = \begin{bmatrix}\eta \\ 0\end{bmatrix}, \tag{4}$$

the free fermion Hamiltonian density is given by

$$P^-_{\rm free} = \eta^\dagger \frac{-(\partial^\perp)^2 + m^2}{i\partial^+}\eta. \tag{5}$$

We note that the fermion mass enters the free Hamiltonian as m^2 and gamma matrices do not appear in this case. There is an explicit chiral symmetry breaking term in the interaction part of the Hamiltonian which is linear in the quark mass and is given by

$$gm\eta^\dagger\sigma^\perp \cdot \left(A^\perp \frac{1}{\partial^+}\eta - \frac{1}{\partial^+}A^\perp\eta\right). \tag{6}$$

Since in the chiral limit we need to avoid degenerate pion and rho, it is clear that we need noncanonical terms in our Hamiltonian that explicitly violate the chiral symmetry and survive the chiral limit. At present investigations are under way to study this problem.

3. High energy scattering

It is well known that the various structure functions one encounters in deep inelastic scattering are Fourier transforms of equal x^+ correlation functions and in the gauge $A^+ = 0$, they are amenable to very clear physical interpretation which leads to the celebtrated parton picture. Also, light-front power counting which is based on light-front symmetries treat x^- and $x^\perp$ differently which is natural for deep inelastic processes. Recently we have attempted to combine coordinate space techniques (Bjorken-Johnson-Low (BJL) expansion plus light-front current algebra) with momentum space techniques (Fock expansion plus ultra-violet renormalization) to address problems at the interface of soft and hard physics. The former leads to bilocal form factors and the later utilizes multi-parton wave functions. The aim is to unify the description of both perturbative and nonperturbative physics using the same language, that of multi-parton wave functions.

As an example, consider the twist two part of the structure function F_2. Utilizing BJL expansion and light-front current algebra one arrives at

$$\frac{F_2(x)}{x} = \frac{i}{4\pi}\int d\xi e^{-i\xi x}\,\overline{V}_1(\xi) \tag{7}$$

where $\xi \doteq \frac{1}{2}P^+y^-$. The bilocal form factor

$$\overline{V}_1(\xi) = \frac{1}{2iP^+}\langle P|\left[\overline{\psi}(y)\gamma^+\psi(0) - \overline{\psi}(0)\gamma^+\psi(y)\right]|P\rangle\cdot \tag{8}$$

Considering a meson like state for the target, we expand the state $|P\rangle$ in terms of the quark-antiquark amplitude Ψ_2, quark-antiquark-gluon amplitude Ψ_3 *etc.* A straight forward evaluation leads to

$$\frac{F_2(x)}{x} = \sum\int|\Psi_2|^2 + \sum\int|\Psi_3|^2 + \cdots \tag{9}$$

Utilizing the fact that the state $|P\rangle$ obeys the eigen value equation, the high energy limit of the structure function, can be computed perturbatively from the knowledge of the high momentum behaviour of multi-parton wavefunctions. In this approach we have investigated [8] various issues, namely, suppression of coherent effects at high energy, cancellation of collinear singularities, emergence of factorization, *etc*. We have also clarified the parton interpretation of the bad ($\perp$) component of the bilocal vector current [9] and shown the important of quark mass in the computation of the transverse polarized structure function in perturbative QCD [10].

4. Open problems

Instead of a summary we list some of the immediate open problems in the field. In order to probe the fate of logarithmic confinement one has to study higher orders in SR scheme. Since the logarithmic confinement in second order is not rotationally invariant, one has to see whether and how rotational symmetry is restored by higher order corrections to the effective Hamiltonian. The study of chiral symmetry breaking on the light-front is in its infancy. One has to study the origin and role of non-canonical operators and their renormalization. The phenomenological consequence of such operators are also worth investigating. Regarding the program for high energy scattering the crucial question is : Can one consistently calculate ? To answer this question, of course, we need to compute higher orders in the BJL expansion. This is especially important for the study of higher twist observables.

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